Predict!

Teaching statistics using informal statistical inference

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Prediction. People engage in prediction everyday. Even though accurate values are unknown, we are not just making wild guesses when we predict. We base predictions on information we already have. If we want to predict who will win the Australian Open, we consider players' past records in other tennis matches and additional contextual information we know (like recent injuries). To predict how much time it will take to drive to the airport, we weigh up our previous experiences going to the airport, the anticipated traffic for that time of day, the distance to the airport and then possibly add some margin depending on the variability we might expect from heavy traffic or unexpected road works—and whether it would be disastrous to arrive late. We do not expect our predictions to be exact or even always correct, but we try and maximise the likelihood that they are.

Statistics is one of the most widely used topics for everyday life in the school mathematics curriculum. Unfortunately, the statistics that we teach focuses on calculations and procedures before students have a chance to see it as a useful and powerful tool. Researchers have found that a dominant view of statistics is as an assortment of tools (calculations and graphs), with few seeing it as a means to understand a complex world (Rolka & Bulmer, 2005).

In this article, *informal statistical inference* is introduced as an approach to teaching statistics. This idea has now been researched from primary school through university classrooms around the world (Arnold, Pfannkuch, Wild, Regan, & Budgett, 2011; English, 2011; Garfield & Zieffler, 2012; Makar, in press). Informal statistical inference can help students better appreciate the usefulness of statistics for both everyday life and future careers. In the next section, informal statistical inference will be introduced and how it differs from the way we usually teach statistics. Next, a unit from a middle school classroom will be used to illustrate how the class were learning statistics while making informal inferences. Finally, some ideas will be provided for turning a regular statistics lesson into one that lets your students make inferences.

Describing data and inferring beyond data

Prediction and estimation are at the core of statistics. Statistical inference is one of the big ideas in statistics, but formal applications of inference (hypothesis testing, parameter estimation) are highly complex and usually not taught until university. Even university students, professionals and researchers find formal statistical inference very challenging to understand and apply appropriately (Erickson, 2006).

Much of the classical content that we teach in school statistics now is *descriptive statistics*. As the name implies, descriptive statistics includes tools for describing data distributions—like averages or range—and ways of representing data, like histograms and box plots. But the power of statistics lies in the ability to go beyond the data we have to make inferences: predictions and estimates about data we do not have. It involves acknowledging the likelihood that comes with the uncertainty of inference (Burrill & Biehler, 2011). An informal statistical inference is a claim (a conclusion such as a prediction, estimate or generalisation) with three characteristics:

- it is aimed at an event beyond the data;
- it is based on data as evidence; and
- it is expressed with uncertainty (Makar & Rubin, 2009).

In other words, an informal statistical inference uses data to make a prediction or conclusion about an uncertain event. An inference must be stated with uncertainty because the exact answer is not known for sure, but can only be estimated. Because inference is where the power of statistics lies, teaching statistics with inference gives students an early and familiar experience with statistics as a powerful tool for investigating the world. Let us look at a middle school classroom that was teaching students about making inferences from data about flight.

Investigating loopy airplanes

The unit described here comes from a suburban state school with students enrolled representing a wide range of performance levels, including a number of students who required substantial learning support. In this classroom, investigations like this were run as guided inquiries (Makar, 2012), which blended direct teaching with opportunities for students to investigate ideas in small groups. The students were not streamed for mathematics, but inquiry units were designed to allow all levels to engage with the inquiry question and have access to powerful statistical ideas.

The teacher used whole class discussions and questioning to seek students' ideas, regularly refocused them on the inquiry question and then capitalised on opportunities to teach or review statistical concepts as they were needed. She gave students time in small groups to plan ways to put the ideas into practice (which were then shared with the class), and circulated to listen to groups and provide support when needed (Allmond, Wells & Makar, 2010).

Figure 1. Two loopy airplanes.

Testing a hunch

The teacher posed the question to the class to investigate: What is the best design for a loopy airplane? A loopy airplane is an aircraft created from a straw and two strips of paper (Figure 1). After the teacher worked with them to negotiate what a 'best' design might mean, the students used data to investigate and make inferences about the designs of loopy airplanes based on how far they flew. The first task was to get a feel for the context. The teacher asked them each to make a loopy airplane with one small loop (made from a 1 cm × 5 cm strip of paper) and one large loop (1 cm × 8 cm strip). While practicing, the class discussed how sometimes the plane flew farther with the small loop in front and sometimes with the larger loop first. To test whether it mattered which loop was in front, the students flew their planes twice each way, measured the distances and collated the data (Figure 2).

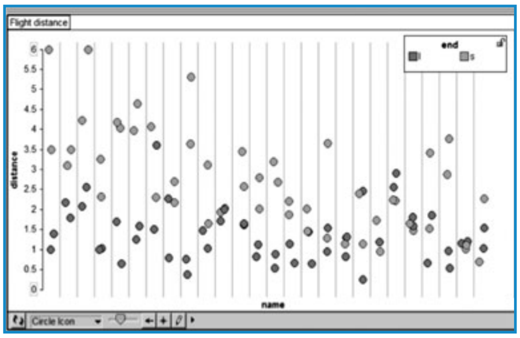

Figure 2. Data from flight distances (in metres) of the loopy planes with small loop first (upper dots) or large loop first (lower dots). Graph created in *TinkerPlots* (Konold & Miller, 2005); student names removed.

The teacher knew that even without any calculations, the students could make an inference from this data. They would not just be describing the data from these specific flights, they would be drawing a general conclusion that went beyond this data about the flight of the loopy planes (in general). From the graph in Figure 2, the teacher asked them whether they could make any conclusions about whether loopy planes likely fly farther when the small loop or the big loop was in front. They would not be able to state their inference in absolute terms (she used the word 'likely' to articulate this) because they could not say for sure what would happen for new flights. Their inference stated a claim (loopy planes likely fly farther if the small loop is in front) that went beyond the data (about flights of the loopy planes in general, not just a description of these flights). In discussing their observations, they were fairly confident that if they flew the loopy planes again, the data would probably lead them to the same conclusion.

Altering the design

In the next lesson, the teacher asked them to test the influence of other possible variables and design new planes by altering the length of the loop (short, medium or long), the width of the loop (narrow, medium or wide) and the placement of the loops (on the ends, in the middle or split—one middle, one end). Because there were 27 combinations (3 lengths × 3 widths × 3 placements), she had each student make and fly their own unique plane (Figure 3).

There was a lot of variability in its flight, even with the same 'thrower', so after a teacher-led discussion, they realised it would be better to collect data from more than one throw of each plane. They collected data on five flights for each plane and averaged them to get a better estimate of each loopy plane's 'typical' distance flown. The teacher helped them to establish protocols for measuring and flying the planes to try and reduce potential sources of variability and error.

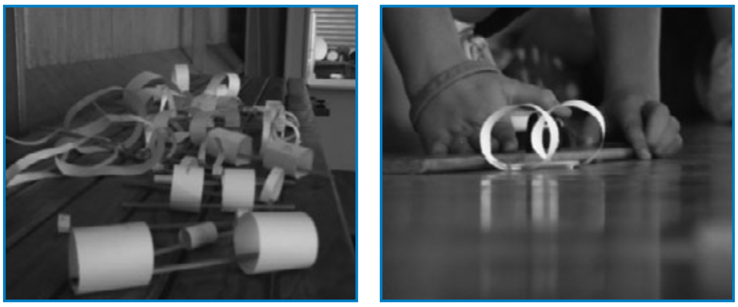

Figure 3. Variations on designs for loopy planes (left) and measuring how far they flew (right).

Analysing the data

Next, the teacher had students enter the data for each plane into *Tinkerplots*. Students used their analysis (Figure 4) as evidence to predict the best aircraft design based on the three variables they investigated:

- best wing width;
- best wing size (length of strip); and
- best wing location.

From the graphs of the average flight for each plane, they noticed that wings that were thin or placed in a split location were quite unpredictable (high variability). They concluded that:

- thin wings tended to fly farther than medium or wide wings (Figure 4, top);

- short-strip wings (smallest) tended to fly farther than medium or long-strip wings (Figure 4, middle); and
- split wings (despite their unpredictability) tended to fly farther than wings placed in the middle or ends of the straw (Figure 4, bottom).

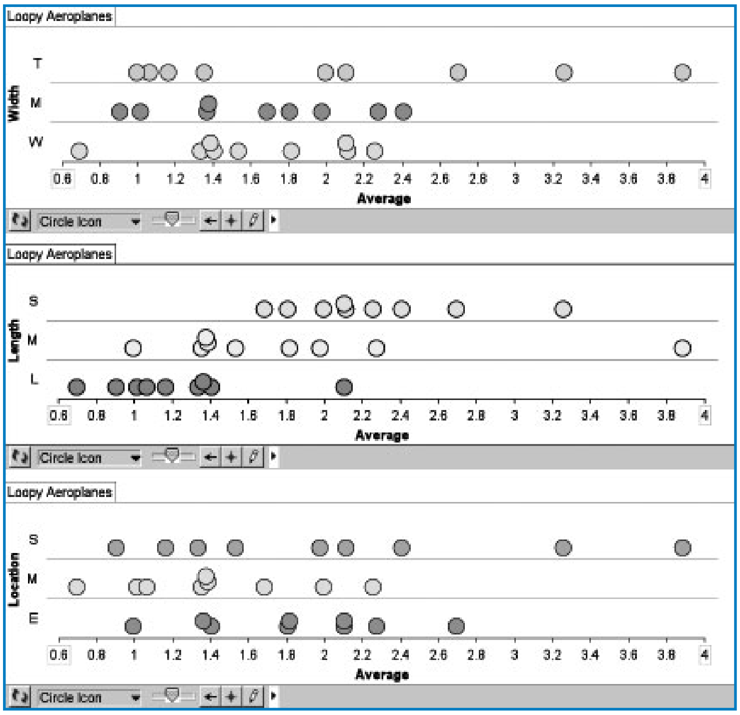

Figure 4. Data from flights (in metres) of 27 different loopy airplanes: Comparing width (top), length (middle) and location of loops (bottom) for each aircraft.

Drawing a conclusion

Their conclusion therefore was that the best design for a loopy airplane was likely the one with thin wings made with short strips of paper and placed in split position on the straw, and would typically fly around 2 metres since the median flight for each of these categories (thin, short strip, split wing position) was around 2 metres. In their whole class reflection at the end of the unit, they recognised that this conclusion was not certain, as it was based only on the data they had collected. In fact, they found it interesting that the individual loopy plane that flew the farthest (3.9 metres) did not have this design!

While students used descriptive statistics in this unit (calculated averages, graphed data as dot plots, compared variability in distributions) as evidence for their inferences, the focus was not on describing the data, it was on using the data to make an inference, or prediction, about which loopy plane design would fly the furthest. They also discussed ideas informally that would not be taught until later years. For example, they debated whether means or medians would be more appropriate because of the variability of the data; and because of the variability in flights, they also thought that collecting more data might make them more confident in their conclusions.

One might argue that the students' flight investigation and analysis had some shortcomings from a professional standpoint. However, the statistical ideas they applied were negotiated throughout the unit using the data as evidence, allowing them to gain an informal understanding of several big ideas in statistics (variability, measures of centre, distribution, sample size, fair testing and inference) (Watson, 2006) at a level appropriate for their age. The concepts that they were exposed to in these lessons could then be built on in later years. In the meantime, they came away from the unit with a sense of the utility of statistics for solving problems of interest to them and a basic understanding of important statistical ideas (Ainley & Pratt, 2010).

Teaching statistics through informal inference

The statistics topics that we teach at school were selected as concepts that would be needed in later years for formal statistical inference, but simple enough so that children would be capable of learning them. This is a 'top-down' view of curriculum design: choosing the topics and concepts to be learned in school statistics based on simplified versions of what is later needed by professionals (Konold,

2007). However, Konold has argued that when we give students the opportunity to work with and analyse data in meaningful ways, they have surprised us with what they are able to understand. From these observations, researchers have studied ways of designing student learning environments through a 'bottom-up' approach that "takes into account not only where we want students to end up, but also where they are coming from" (p. 270). Informal statistical inference is one of these concepts. It engages students in statistical ideas that they will need to learn, but takes into account their everyday experiences with prediction and inference. As argued by Seymour Papert (2006, p. 585), "Instead of making children learn the math they hate let's make a mathematics they will love". This is critical if we are to reverse the trend of declining enrolments in mathematics (McPhan et al., 2008).

In the flight example above, students experienced all three characteristics of statistical inference (Makar & Rubin, 2009). At the end of their investigation, they drew a conclusion (claim) that:

- was aimed at the design and flight distance of loopy airplanes in general (an event beyond just the data they collected);
- was based on their analysis of flight data (data as evidence); and
- used the words "likely" and "typically" in their conclusion to recognise that they could not be certain of their answer since it was only based on the data they collected (expressed with uncertainty).

Making statistical questions inferential

To give students an opportunity to make inferences, many of the current questions asked in statistics lessons could be easily amended by adding an additional question about an unknown context. Knowledge of the context may require an adjustment. Several examples have been listed in Table 1.

Table 1. Making descriptive questions more inferential. Assume data were provided.

Descriptive version	Inferential addition
What are the most common lengths of the names of students in our class?	**Add**: What do you predict would be the most common lengths of names in our community?
What proportion of students in our class like pepperoni pizza?	**Add**: What would you estimate to be the proportion of students in the class next door who like pepperoni pizza?
A shower timer is considered accurate if it is within 10% of its 4-minute target. Calculate the number of shower timers in the list below that are accurate.	**Add**: What proportion of shower timers would you expect to be accurate coming from the same factory?
Graph the data for 10 jump lengths of an origami frog.	**Add**: How far does an origami frog jump (in general)?
Find the average fuel consumption for twenty 2013 models of small cars driven at 80km/hour.	**Add**: What do you anticipate the average fuel consumption to be for 2014 models of small cars driven at the same speed? [Students may choose to lower the fuel consumption in recognition of possible technological improvements.]

In each of these examples, students would be asked to make an inferential statement that:

- applied to a more general situation beyond the data they had available;
- required them to use the data they had (and calculations or graphs that helped to analyse or interpret their data) as evidence for their inference; and
- stated their inference with some uncertainty.

You would encourage students to recognise, for example, that the most common lengths of names in the community would not be exactly the same as their class, so they may want to use a range of values in their estimate (rather than a single

value) and express their answer with some uncertainty. These simple adjustments also encourage students to consider the data as a whole when making predictions, rather than focus on individual points. The focus on data as an aggregate is a key concept—and difficulty—for learners in statistics (Konold & Pollatsek, 2002; Watson, 2006). In addition, making inferences allows students to bring in their own contextual knowledge in ways that are relevant.

Building informal inference into statistical inquiry

You can build inference into ordinary statistics questions (as in Table 1) or you can give students more inferential experiences by engaging them in statistical inquiry. The flight unit described earlier is one such example. Below is a list of statistical inquiry questions that have come out of middle school classrooms in Australia (see also Allmond et al., 2010; Fielding-Wells, 2010; 2013).

- Do we eat healthy cereal for breakfast?
- Do shower timers really last for four minutes?
- How long does it take to walk 10 000 steps?
- How long does it take to read a book?
- How much do we typically grow in primary school?
- Are athletes getting faster over time?
- What are the characteristics of a good handball player?
- What songs should we play for the school disco?
- Is it better to make or buy take away chinese food?
- Does Barbie have human proportions?
- What is your reaction time?
- How far does an origami frog jump?
- What fraction of the newspaper is ads?

In statistical inquiry, students go through the whole statistical investigation cycle of posing a question, planning their investigation, collecting and analysing data and drawing a conclusion (MacGillivray & Pereira-Mendoza, 2011; Wild & Pfannkuch, 1999). They also build critical and creative thinking, scepticism, curiosity, argumentation, collaboration and ways to manage ambiguities that cannot be captured by textbook-type problems.

Conclusion

Statistics is different than mathematics in that conclusions do not necessarily follow deductively. Statistics is purpose-built for addressing uncertainty and variability. However, the skills, techniques and procedures often taught in school do not take advantage of the power of statistics in engaging with uncertain and unknown situations. With the introduction of the *Australian Curriculum: Mathematics*, there is an opportunity to reflect on and change the way we teach statistics at all levels. The overall content descriptors in probability and statistics call for students to be able to "recognise and analyse data and draw inferences" (ACARA, 2013). There is a renewed focus on problem-solving and reasoning that are at the heart of informal statistical inference. By developing students' experiences in making informal statistical inferences, students not only build and apply the statistical concepts that are in the curriculum, but also statistical ways of thinking and first-hand appreciation for the relevance of statistics for the predictions, estimates and data-based conclusions they already make in their everyday life. So rather than teach only statistics, add just a dash or a whole bucket of prediction by including informal inference in your lessons.

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